Estimates for the spatial locations of the indoor objects by radial distributions

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Abstracts

Recently we daily use the global positioning systems for obtaining the location for car navigation. These systems are very convenient for driving, but we sometimes need more accurate system for obtaining the locations of the specified objects like sports athletes, and also may demand for the location estimation in the indoor environments for obtaining the nursing care information in hospitals. We propose the statistical method for estimating the location in the room where we cannot receive the satellite information on the location. We use the time of arrival (TOA) data based on the ultra-wideband (UWB) tag system. The proposed method is based on the marginal likelihoods of radial distributions generated by positive survival distribution among the several anchor radio sites placed in the room. We shall compare the proposed statistical method and other previous methods and conclude that our iterative method is promising for practical use.

Keywords: Radial distribution, spatial location estimates, global positioning system, marginal distribution

1. Introduction

Recently we are required to contrive the location estimating system for the target object using the ultra-wideband (UWB) tags. The first UWB communications patent was issued in 1973 to Gerald F. Ross on transmission and reception of baseband pulse signals (Sahinoglu, Gezic, and Guvenc, 2008; Ross, 1973). In this article we would like to estimate the spatial location based on the distance data of the transmitter and the receiver. The data set is called Time-of-Arrival data (ToA data) and obtained by measuring the distance between the TAG (receiver) and the ANCHOR (transmitter). The radio wave is sent at time $t_0$ and it is received by ANCHOR at time $t_1$, and then the distance will be estimated $c(t_1-t_0)$, where $c$ is the light speed. Our goal is to estimate TAG position using ToA data. If the distances between the TAG and several ANCHORs are estimated without observation errors, no difficult problems are occurred. We can obtained the goal just by calculating the position based on the triangulation method. The ToA data set includes the larger errors owing to reflections of waves against walls and other disturbance noises.

2. The statistical Model

We have two types of data set; one is the data set when the subject with the TAG keeps stopping within a minute on the fixed point. The other is the data set when the subject with the TAG moves on the prescribed trajectory. The latter is, of course, extended to much difficult situation, in case that the subject is randomly walking in the space. However, we do not consider this situation in this paper because of the evaluation of the estimation method. Watabe and Kamakura (2010) proposed a convenient and very fast algorithm with compared to other works (Carrery, 2000, Venk and You, 2004; Nanotoron, 2007; Yanagi, 2009). This algorithm is very fast and gives the better estimates, but the estimates are just point estimates. In this article we consider the confidence regions of TAG position based on the new statistical model described below.
Considering that the obtained distance data are all positive, we propose the following circular distribution generated by Weibull distribution.

\[
g(x, y) = \frac{\lambda m}{2\pi} \left( x^2 + y^2 \right)^{m-1} \exp \left\{ -\lambda \left( x^2 + y^2 \right)^{\frac{m}{2}} \right\} \quad (x, y, m, \lambda > 0)
\]  

(1)

This distribution has very interesting property that includes bivariate normal distribution as the special case as described the following Fig. 1. The Fig. 2 is the standard case of the above density that depicts the volcano-like density with hole in the center.

Fig. 1: Bivariate Normal ( \( m = 2, \lambda = 0.5 \) )

Fig. 2: Vocalno like density \( m = 3, \lambda = 0.5 \) ( \( m = 3, \lambda = 0.5 \) )

Assuming that each base station observers independent measurements, the likelihood based on the data set is calculated as follows:

\[
L(\lambda_1, m_1, ..., \lambda_K, m_K) = \prod_{i=1}^{K} \prod_{j=1}^{n_i} \frac{\lambda_i m_i}{2\pi} \left\{ (x_{ij} - c_{i1})^2 + (y_{ij} - c_{i2})^2 \right\}^{\frac{m_i}{2}-1} \\
\times \exp \left[ -\lambda_i \left\{ (x_{ij} - c_{i1})^2 + (y_{ij} - c_{i2})^2 \right\}^{\frac{m_i}{2}} \right] 
\]

(2)

Here \( K \) is the number of stations and for each station \( i \) the sample size is \( n_i \). The observed data set for station \( i \) is \((x_{ij}, y_{ij})\). The coordinates \((c_{i1}, c_{i2})\)’s are given station positions.
3. Approximate MLE

We must estimate the parameters for each base station based on the marginal or conditional information on distance measurements. Here we note that the conditional distribution \((x>0, y=0)\) and the marginal distribution \(r\) are different. The Fig. 3 illustrates this.

The differences between conditional and marginal distributions are getting smaller for the large shape parameter \(m\). Convenient adjustment is adding one for the obtained marginal likelihood estimate.

Now numerical calculations give us the ML estimates for each station parameters. We calculate the estimates of \(x, y\) location parameters by maximizing the joint estimated likelihoods:

\[
g(\alpha, \beta) = \prod_{i=1}^{K} \hat{f}_{i}(\alpha, \beta) = \prod_{i=1}^{K} \frac{\hat{\lambda}_i \hat{m}_i}{2\pi} \left\{ (\alpha - c_{i1})^2 + (\beta - c_{i2})^2 \right\}^{\frac{\hat{m}_i}{2}} \times \exp \left\{ -\hat{\lambda}_i \left\{ (\alpha - c_{i1})^2 + (\beta - c_{i2})^2 \right\}^{\frac{\hat{m}_i}{2}} \right\} (3)
\]

The MLE of the TAG location is obtained from solving numerically the following simultaneous equations:

\[
F = \frac{\partial \log g}{\partial \alpha} = \sum_{i=1}^{K} \left[ \frac{(\hat{m}_i - 2)(\alpha - c_{i1})}{(\alpha - c_{i1})^2 + (\beta - c_{i2})^2} - \hat{\lambda}_i \hat{m}_i (\alpha - c_{i1}) \left\{ (\alpha - c_{i1})^2 + (\beta - c_{i2})^2 \right\}^{\frac{\hat{m}_i}{2}} \right] = 0
\]

\[
G = \frac{\partial \log g}{\partial \beta} = \sum_{i=1}^{K} \left[ \frac{(\hat{m}_i - 2)(\beta - c_{i2})}{(\alpha - c_{i1})^2 + (\beta - c_{i2})^2} - \hat{\lambda}_i \hat{m}_i (\beta - c_{i2}) \left\{ (\alpha - c_{i1})^2 + (\beta - c_{i2})^2 \right\}^{\frac{\hat{m}_i}{2}} \right] = 0
\]

The asymptotic variance of the \(x\)-location is as follows:
\[
AVar\left[\alpha(\hat{m}_1, \hat{\lambda}_1, \cdots, \hat{m}_K, \hat{\lambda}_K)\right] = \mathbf{h}_{11}^T \hat{\Sigma} \mathbf{h}_{11}
\]

\[
\mathbf{h}_{11} = \left( \frac{\partial \alpha}{\partial \hat{m}_1}, \frac{\partial \alpha}{\partial \hat{\lambda}_1}, \cdots, \frac{\partial \alpha}{\partial \hat{m}_K}, \frac{\partial \alpha}{\partial \hat{\lambda}_K} \right)
\]

(5)

Here we note that the tedious calculations are needed for the differentiations by the theorem on implicit functions for maximization. Similarly, asymptotic variance of the \(y\)-location is as follows:

\[
AVar\left[\beta(\hat{m}_1, \hat{\lambda}_1, \cdots, \hat{m}_K, \hat{\lambda}_K)\right] = \mathbf{h}_{22}^T \hat{\Sigma} \mathbf{h}_{22}
\]

\[
\mathbf{h}_{22} = \left( \frac{\partial \beta}{\partial \hat{m}_1}, \frac{\partial \beta}{\partial \hat{\lambda}_1}, \cdots, \frac{\partial \beta}{\partial \hat{m}_K}, \frac{\partial \beta}{\partial \hat{\lambda}_K} \right)
\]

(6)

Then, the asymptotic covariance matrix becomes

\[
ACov[(\hat{\alpha}, \hat{\beta})] = H^T \hat{\Sigma} H
\]

\[
H^T = \begin{pmatrix}
\frac{\partial \alpha}{\partial \hat{m}_1} & \frac{\partial \alpha}{\partial \hat{\lambda}_1} & \cdots & \frac{\partial \alpha}{\partial \hat{m}_K} & \frac{\partial \alpha}{\partial \hat{\lambda}_K} \\
\frac{\partial \beta}{\partial \hat{m}_1} & \frac{\partial \beta}{\partial \hat{\lambda}_1} & \cdots & \frac{\partial \beta}{\partial \hat{m}_K} & \frac{\partial \beta}{\partial \hat{\lambda}_K}
\end{pmatrix}
\]

(7)

Here we note that the following notation should be used.

\[
\frac{\partial \alpha}{\partial \hat{m}_i} = \frac{D(F, G)}{D(\beta, \hat{m}_i)} / \Delta \\
\frac{\partial \alpha}{\partial \hat{\lambda}_i} = \frac{D(F, G)}{D(\beta, \hat{\lambda}_i)} / \Delta \\
\frac{\partial \beta}{\partial \hat{m}_i} = \frac{D(F, G)}{D(\alpha, \hat{m}_i)} / \Delta \\
\frac{\partial \beta}{\partial \hat{\lambda}_i} = \frac{D(F, G)}{D(\alpha, \hat{\lambda}_i)} / \Delta
\]

(\text{for } i=1, \ldots, K)

\[
\Delta = \frac{D(F, G)}{D(\alpha, \beta)}
\]

The above results come from the theorem on implicit functions. Finally we can obtain the confidence region by the following inequality equation.

\[
(x - \hat{\alpha}, y - \hat{\beta})\left[ H^T \hat{\Sigma} H \right]^{-1} \begin{pmatrix}
x - \hat{\alpha} \\
y - \hat{\beta}
\end{pmatrix} \leq \chi^2_{p}(p)
\]

(8)

The asymptotic covariance matrix needed for calculation of (8) is as follows:

\[
\Sigma = \begin{pmatrix}
I_1^{-1} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & I_K^{-1}
\end{pmatrix}
\]
\[ I_i^{-1} = \frac{6}{n,\pi^2} \begin{pmatrix} m_i^2 & m_i \lambda_i (\gamma + \log \lambda_i - 1) \\ m_i \lambda_i (\gamma + \log \lambda_i - 1) & \lambda_i^2 \left[ 1 + (\gamma - 2)\gamma + \pi^2 + 2 \log \lambda_i (\gamma + \log \lambda_i - 1) \right] \end{pmatrix} \]

4. Some simulation results
Assuming the distance from the true TAG location \((\alpha_0, \beta_0)\) with Weibull, we generate the independent sample for each base station.

\[ \ell^2 = (\alpha_0 - c_{11})^2 + (\beta_0 - c_{12})^2 \]
\[ r_i \sim \text{Weib}(\text{shape} = m, \text{scale} = \frac{\ell_0}{\Gamma(1 + 1/m)}) \]

We can just show you the case of \((\alpha_0, \beta_0) = (0.25, 0.25)\).

Fig 4: Simulated contour graphics of the joint estimated likelihood

5. Conclusions
The doughnut distribution (Weibull circular distribution) generated from the Weibull distribution is useful for estimating the TAG locations. The doughnut distribution with shape parameter becomes 2-dimensional normal distribution. We can estimate parameter based on the marginal distribution from doughnut distribution. We proposed the new method for estimating the TAG locations with confidence regions. Further investigations for real data are needed and fast calculation method for maximizing the joint probabilities is also required. The above results come from the theorem on implicit functions. Finally we can obtain the confidence region by the following inequality equation.

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References


