Multivariate Hierarchical Normal Modelling under Informative Sampling

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In this paper a model-dependent approach for multivariate hierarchical normal modelling that accounts for informative probability sampling of first and second level population units is developed. The proposed approach consists of first extracting the hierarchical model holding for the sample data given the selected sample as a function of the corresponding population model and the sample selection probabilities, and then fitting the resulting sample model using Bayesian methods. This approach evolved from similar models developed earlier for univariate responses, which the paper reviews. An application of the approach is presented for modelling jointly Mathematics and Portuguese Language proficiency scores obtained from a Brazilian evaluation study of basic education conducted by the Brazilian National Institute of Education Research (INEP). The scores stem from applying Item Response Theory models to test results from the 'Prova Brasil 2009' study. A two-level multivariate hierarchical normal model is fitted, where the students and schools are respectively the first and the second levels. The analysis is restricted to the performance of students from the 8th grade in elementary schools from Rio de Janeiro municipality. Simulation is also carried out in order to assess the frequentist properties of the approach.

Some key words: Credibility interval; Markov chain Monte Carlo; Probability weighting; Educational assessment.