Quantile Correlations and Quantile Autoregressive Modeling

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In this paper, we propose two important measures, quantile correlation (QCOR) and quantile partial correlation (QPCOR). We then apply them to quantile autoregressive (QAR) models, and introduce two valuable quantities, the quantile autocorrelation function (QACF) and the quantile partial autocorrelation function (QPACF). This allows us to extend the Box-Jenkins three stage procedure (model identification, model parameter estimation, and model diagnostic checking) from classical autoregressive models to quantile autoregressive models. Specially, the QPACF of an observed time series can be employed to identify the autoregressive order, while the QACF of residuals obtained from the fitted model can be used to assess the model adequacy. We not only demonstrate the asymptotic properties of QCOR, QPCOR, QACF, and PQACF, but also show the large sample results of the QAR estimates and the quantile version of the Ljung-Box test. Moreover, we obtain the bootstrap approximations to the distributions of parameter estimator and proposed measures. Simulation studies indicate that the proposed methods perform well in finite samples, and an empirical example is presented to illustrate usefulness.

Key Words: Autocorrelation function; Bootstrap method, Box-Jenkins method, Quantile correlation, Quantile partial correlation, Quantile autoregressive model