

Bayesian Estimation of the Spatial Variation of the Completeness Magnitude for the Venezuelan Seismic Catalogue

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Abstract

We apply Bayesian inference in order to assess the completeness magnitude M_c of an instrumental seismological catalogue compiled by the Venezuelan Foundation for Seismological Research during 2000-2010. The catalogue consist of 18774 well localized earthquakes. The events registered in this catalogue have been detected by the Venezuelan Seismological Network. The earthquakes are analyzed under the approach known as the Bayesian magnitude of completeness (BMC) method. In particular, we explore the spatial variation of M_c depending on the minimum number of stations that detected the seismic event and the distribution of seismic stations. We explore this technique to characterize the quality of the catalogue, to perform high resolution M_c mapping and to minimize errors in the M_c estimates due to spatial heterogeneities.

Key Words: earthquakes, Bayes' theorem

1. Introduction

North Venezuela is part of the east-west oriented boundary zone between the Caribbean and the South American plates (figure 1). This complex deformation zone is 200 to 300 km wide (Russo and Speed, 1992). The Caribbean plate moves eastward relative to the South American plate with a dextral relative movement calculated of about 23 mm/a (Deng and Sykes, 1995). This plate boundary has generated a strike slip right lateral active fault system, which extends from the Venezuelan/Colombian border to the northeastern edge of the continent. This fault system was denominated Boconó- San Sebastián- El Pilar. Also, beside this principal fault system, exist others minor active faults systems such as Oca-Ancón, Valera, La Victoria and Urica (Schubert, 1981).

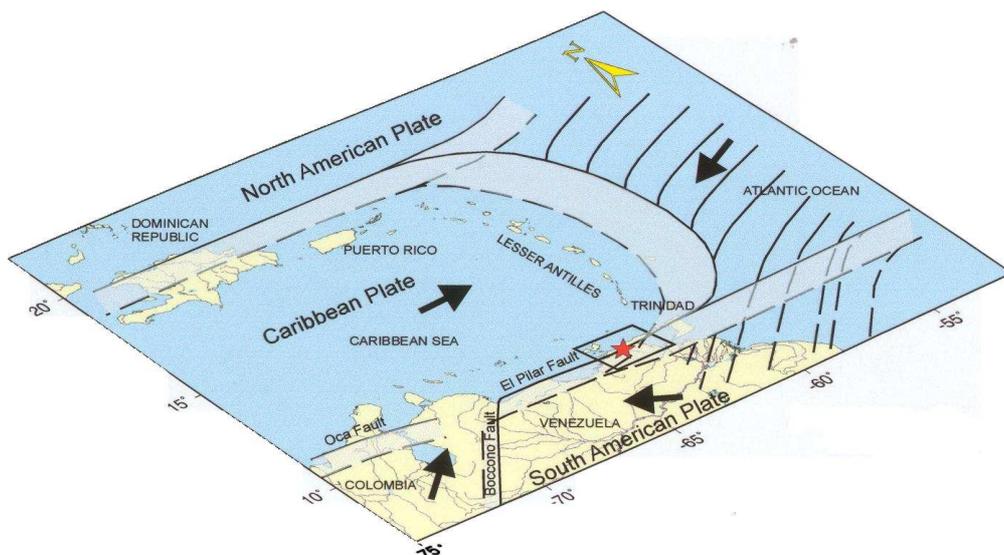


Figure 1: Tectonic map of Venezuela (Baumbach et al., 2001).

Venezuelan Foundation of Seismological Research (FUNVISIS) is responsible for monitoring earthquakes in Venezuela. It consists of a central recording system currently with 35 three-component broadband stations (figure 2). The national network instruments are operating in continuous-recording mode since 2000. The network is equipped with a system of automatic earthquake detection followed by a manual verification. The data recorded by the national network are used to generate monthly seismological bulletins and form a high-quality earthquake catalogue is constantly update.

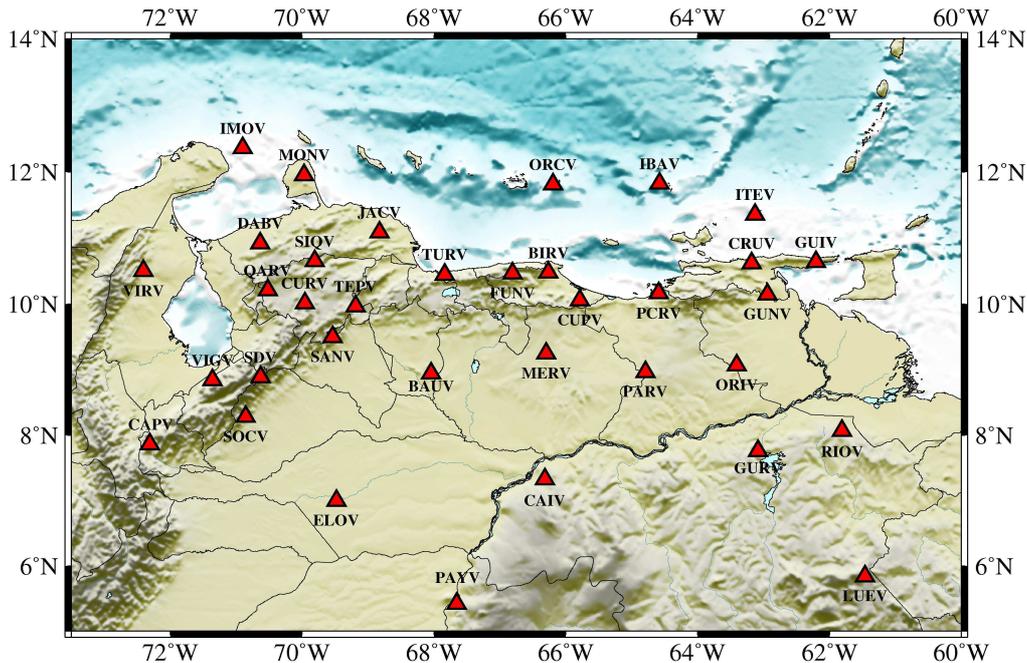


Figure 2. Venezuelan National Network.

2. Earthquake Catalogue

We select a set of 18774 well localized earthquakes in the FUNVISIS catalogue from 1 January 2000 to 21 December 2010 (figure 3). The study is defined for the region between latitudes 5-14 degrees N and longitudes 74-58 degrees W; this area includes Venezuela, The Caribbean Sea, Trinidad and Tobago, Netherlands Antilles and northeast of Colombia. We only consider earthquakes located by at least three seismic stations, with M_w magnitude and RMS less than 0.9.

3. Method

The completeness magnitude (M_c) is defined as the lowest magnitude at which all earthquakes in a space-time volume are reliably detected (Lay and Wallace, 1995). If the magnitudes of a set of earthquakes obey the Gutenberg-Richter law (Gutenberg and Richter, 1944), M_c can also be defined as the minimum magnitude at which the cumulative frequency magnitude distribution departs from the exponential decay. Knowledge of the completeness magnitude is crucial for any seismicity analysis.

In the present work we propose a model to compute M_c in space based on the proximity to seismic stations in a network. We consider a new M_c mapping approach using the Bayesian magnitude of completeness method based on a four-step procedure:

(1) In the entire seismicity region we defined a 22 km two-dimensional grid (figure 4).

(2) We used an empirical relationship $M_c^{\text{pred}} = 4.81d^{0.0883} - 4.36$ ($\tau_0 = 0.19$ the standard deviation) proposed by Mignan et al. (2011) between M_c^{pred} and the distance d to the 3rd nearest station. We computed M_c^{pred} for every particular cell of the grid.

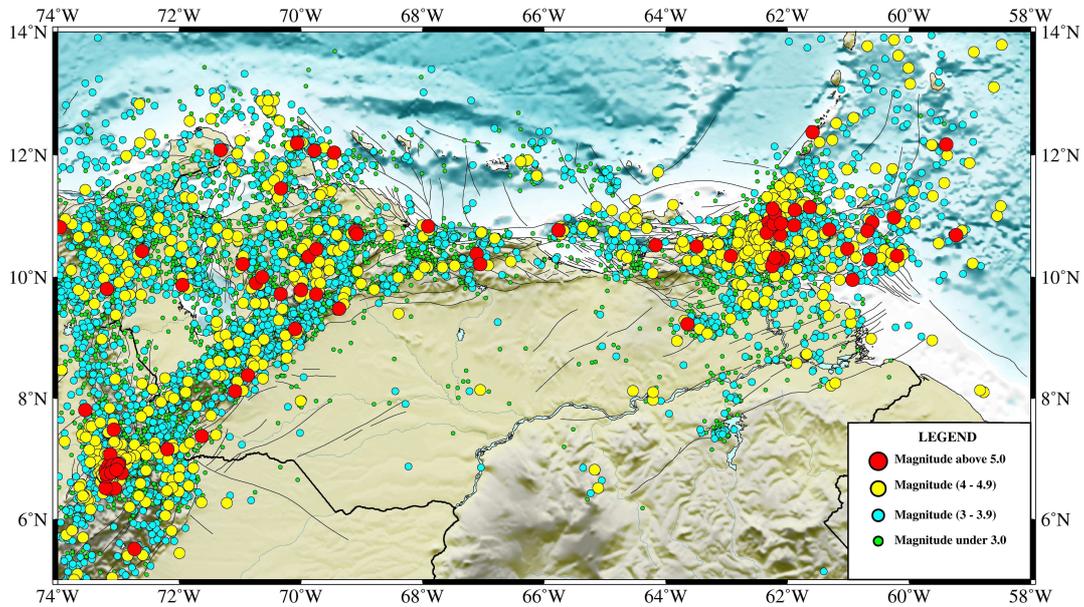


Figure 3. Earthquakes listed in the FUNVISIS catalogue from 2000 to 2010.

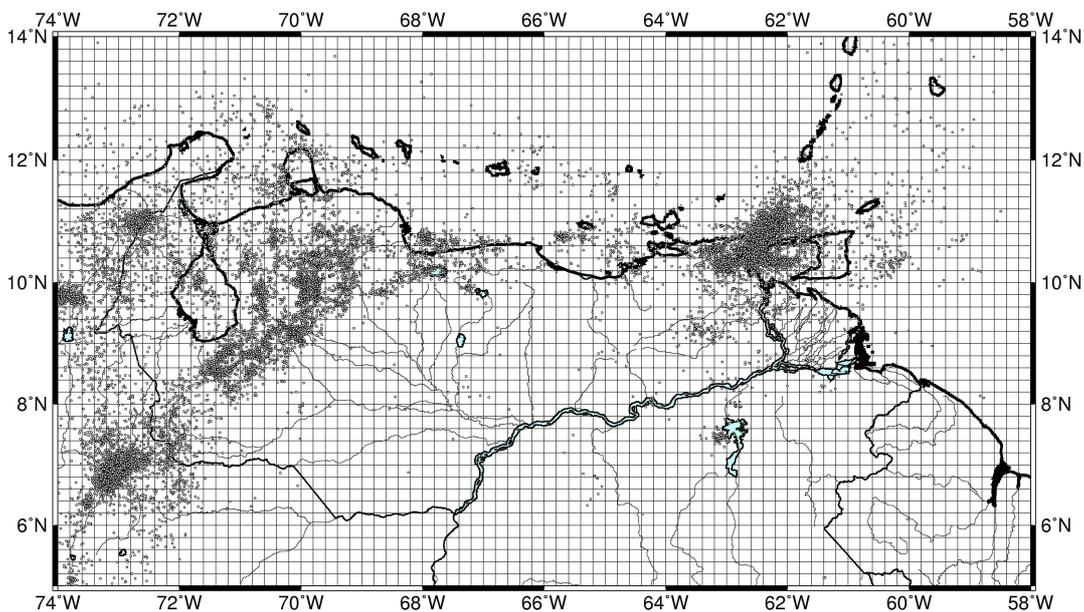


Figure 4. 22 km two – dimensional grids.

(3) We computed M_c^{obs} using the nonparametric maximum curvature method (MAXC) (Wyss et al., 1999) for every particular cell of the grid taking at least 5 earthquakes. MAXC is robust and

simple technique to perform such calculations. The MAXC technique consists of finding the magnitude bin with the highest frequency of events in the frequency-magnitude plot (figure 5). We generate $n = 1000$ Mc^{obs} by using simple random sampling method and then computed mean and variance (equations 4 and 5).

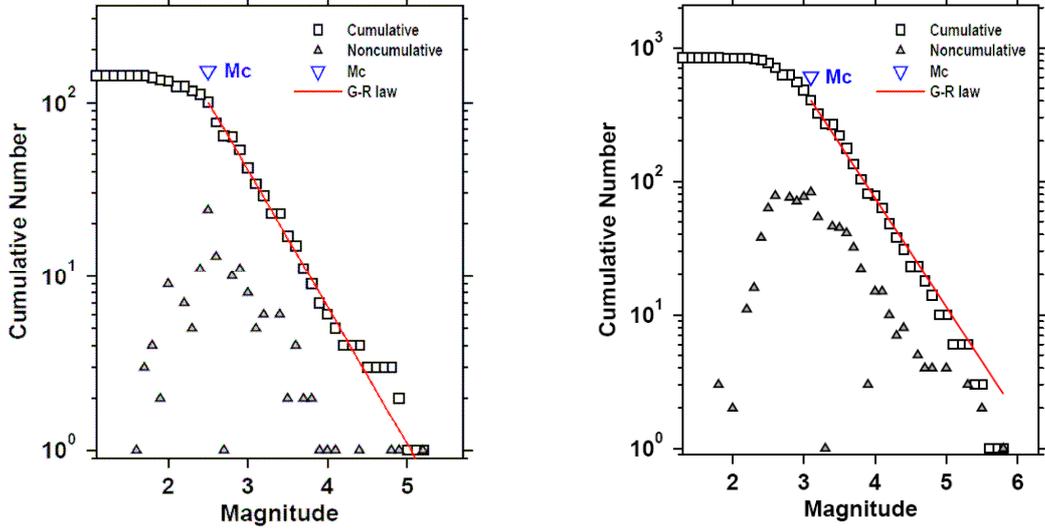


Figure 4. Mc^{obs} computed by using the Maximum Curvature Method (MAXC).

(4) In the Bayesian approach, the observed empirical relationship between the completeness magnitude and the proximity to the seismic stations can be used as a priori information described by the prior probability distribution $p(Mc, \sigma^2)$. Local completeness estimates based on observed magnitudes can be expressed by the conditional data likelihood $p(Mc^{obs} | Mc, \sigma^2)$. The best estimate of the completeness, which employs both the prior information and the local observation, each weighted by their uncertainty, given by the posterior distribution in Bayes' theorem,

$$p(Mc, \sigma^2 | Mc^{obs}) = \frac{p(Mc^{obs} | Mc, \sigma^2) p(Mc, \sigma^2)}{p(Mc^{obs})} \quad (1)$$

Where $p(Mc^{obs})$ is the marginal distribution. The residual between the predicted and the observed completeness magnitude ($Mc^{pred} - Mc^{obs}$) is well approximated by a Gaussian distribution. We therefore define the prior probability distribution as,

$$p(Mc) = \frac{1}{\tau_0 \sigma \sqrt{2\pi}} \exp\left(-\frac{(Mc - M^{pred})^2}{2\tau_0^2 \sigma^2}\right) \quad (2)$$

Where Mc^{pre} was estimated in step 2. We also assume the conditional data likelihood to be (Evans y Rosenthal, 2005) as,

$$p(Mc^{obs} | Mc, \sigma^2) = (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{n}{2\sigma^2} (\overline{Mc}^{obs} - Mc)^2\right) \exp\left(-\frac{n-1}{2\sigma^2} s^2\right) \quad (3)$$

Where,

$$\bar{Mc}^{obs} = \frac{1}{n} \sum_{i=1}^n Mc_i^{obs} \quad (4)$$

The sample mean, and,

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (Mc_i^{obs} - \bar{Mc}^{obs})^2 \quad (5)$$

The sample variance, both were estimated in step 3.

The marginal prior distribution of the variance σ^2 ,

$$\frac{1}{\sigma^2} \sim \text{Gamma}(\alpha_0, \beta_0) \quad (6)$$

With the prior and data likelihood defined as in equations (2) and (3), the posterior equation becomes a normal inverse-gamma distribution,

$$p(Mc, \sigma^2 | Mc^{obs}) \sim N\left(\bar{Mc}, \left(n + \frac{1}{\tau_0^2}\right)^{-1} \sigma^2\right) \quad (7)$$

With,

$$\frac{1}{\sigma^2} \sim \text{Gamma}\left(\alpha_0 + \frac{n}{2}, \beta_x\right) \quad (8)$$

Where,

$$\bar{Mc} = \left(n + \frac{1}{\tau_0^2}\right)^{-1} \left(\frac{Mc^{pred}}{\tau_0^2} + n\bar{Mc}^{obs}\right) \quad (9)$$

And,

$$\beta_x = \beta_0 + \frac{n}{2} (\bar{Mc}^{obs})^2 + \frac{1}{2\tau_0^2} (Mc^{pred})^2 + \frac{n-1}{2} s^2 - \frac{1}{2} \left(n + \frac{1}{\tau_0^2}\right)^{-1} \left(\frac{Mc^{pred}}{\tau_0^2} + n\bar{Mc}^{obs}\right)^2 \quad (10)$$

From equation (4) we see that $p(Mc, \sigma^2 | Mc^{obs})$ equals Mc^{pred} in cell without data, where the uncertainty of the observed value would be infinite. For other cells, the higher the uncertainty of Mc^{obs} estimates, the higher the weight of Mc^{pred} .

4. Results and Conclusions

The results show an overall variation of M_c given by the posterior distribution in Bayes' theorem from 2.3 to 3.9. We found different thresholds and ranges of M_c depending on the dimension of the seismicity zone: western region from 2.4 to 3.6, north central from 2.5 to 3.3 and eastern region from 2.3 to 3.6. We also include remarks in border seismicity, close to Colombia, Trinidad and Caribbean Sea, where the largest M_c values are estimated.

5. References

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